

Nonlinear aspects of an ecological-economic interest conflict in the lake pollution management problem

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Peculiarities of forward looking dynamics

- Economists like to model planning agents, especially perfectly rational agents
- This leads to dynamical systems where the planning agent can choose an initial condition
- The choice is constrained by transversality conditions; usually they impose a bound on the growth of solutions
- Perfect foresight is needed to pick the correct initial condition — in many cases on some (stable) invariant manifold of the dynamical system

Global considerations

- Well-understood for “local” problems around a single steady state
- If there are multiple candidates for long-term steady states, the problem becomes non-local:
 - Ecological systems: e.g. economics of the pollution of lakes
Brock and Starret (2003) & Mäler et.al. (2003)
- **Classification problem** according to qualitative properties of the solutions
- **Bifurcation of Indifference Points** of non-convex optimal control problems

Context

- Moghayer(2011): Developed new methods to analyse non-convex discrete time optimal control problems:
 - obtain the **global** solution structure:
FOCs \rightarrow Dynamical Systems $(\mathcal{X}, \mathcal{I}, \varphi_r^t)$
 - identify qualitative changes of this structure: bifurcation analysis
 - develop numerical geometrical methods
- Distinctive feature: occurrence of *indifference states*
- Genesis of indifference thresholds (Skiba states) through *indifference-attractor bifurcation*
- Present talk: application to lake pollution management problem

Lake as Microcosm

Lakes:

- Mikos-Kosmos reflecting the entire world
- Extensive enough to harbour many scales of complexity
- Just the size!

Shallow Lakes:

- can be largely colonised by rooted submerged vegetation
- can be very important in populated areas



Lake O'Hara and Mary Lake

Discrete time lake model (Mäler et al., 2003)

Shallow lake dynamics

$$x_{t+1} = u_t + (1 - b)x_t + \frac{x_t^q}{1 + x_t^q}, \quad x_0 \text{ given,}$$

- $x_t \geq 0$: amount of phosphorus in the lake;
- u_t : external loading of phosphorus;
- b : natural robustness (rate of loss of phosphorus);
- $\frac{x_t^q}{1 + x_t^q}$: biological production of phosphorus in the lake
 - q : responsiveness of the lake

Shallow lakes

response function for different rate of sedimentation

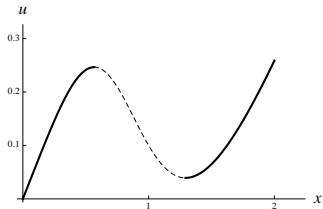


Figure: hysteresis

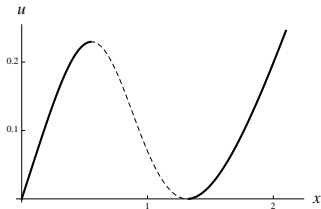
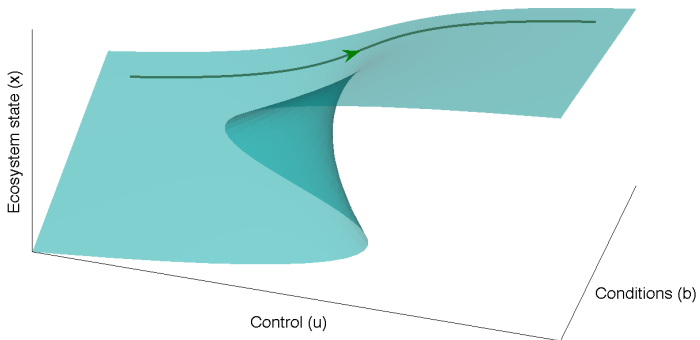
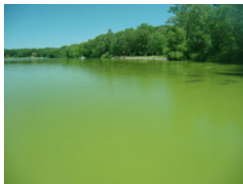
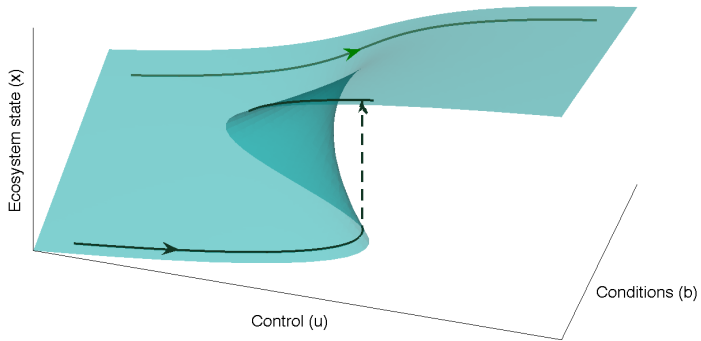


Figure: irreversible

Critical transition in lakes



Critical transition in lakes



Critical transition in lakes

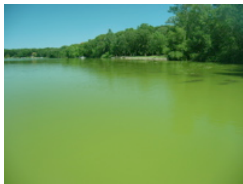
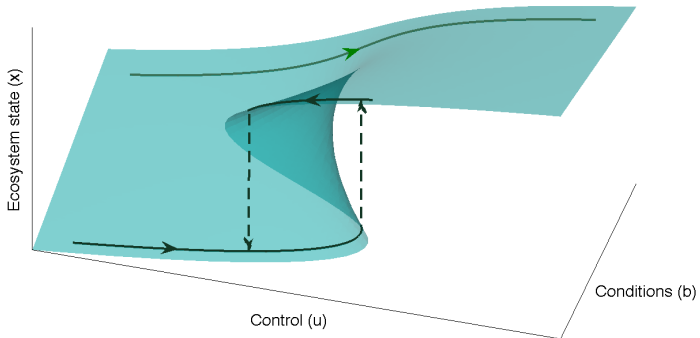


Illustration: stability landscape

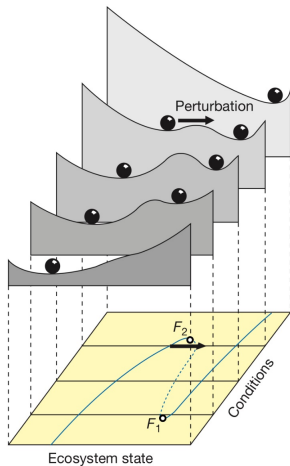


Figure: Scheffer e. al. Nature 2011

Discrete time lake pollution problem (DLP)

(Brock and Starrett (1999); Dechert and Brock (2000); Mäler et al. (2000))

- The social welfare stream : $g(x_t, u_t) = \log u_t - cx_t^2$,
 - $\log u_t$: benefit to farmers from using fertilizers
 - cx_t^2 : weight of interests (cost of pollution to lake users)

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 - $\log u_t$: benefit to farmers from using fertilizers
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- Social planner problem :

$$V(x_0) \equiv \max_{\{u_t\}} \sum_{t=0}^{\infty} (\log u_t - cx_t^2) e^{-\rho t},$$

subject to,

$$x_{t+1} = u_t + (1 - b)x_t + \frac{x_t^q}{1 + x_t^q},$$

- $\rho > 0$: discount rate

DLP economic optimisation problem

Pontryagin's maximum principle

Pontryagin function :

$$P(x_t, u_t, y_{t+1}) = y_{t+1}f(x_t, u_t) + e^\rho g(x_t, u_t).$$

Necessary conditions for optimality :

- $P_{u_t} = 0$, $e^\rho y_t = P_{x_t}$, $x_{t+1} = P_{y_{t+1}}$
- Boundary conditions

From $P_{u_t} = 0$ we obtain

$$u_t = U(y_{t+1}) = -\frac{e^\rho}{y_{t+1}}$$

Discrete Hamilton function:

$$H(x_t, y_{t+1}) = y_{t+1} f(x_t, U(y_{t+1})) + e^\rho g(x_t, U(y_{t+1}))$$

Canonical equations:

$$x_{t+1} = H_{y_{t+1}} = f(x_t, U(y_{t+1})),$$

$$e^\rho y_t = H_{x_t} = y_{t+1} f_x(x_t, U(y_{t+1})) + e^\rho g_x(x_t, U(y_{t+1})).$$

These equations can be solved to yield the phase map

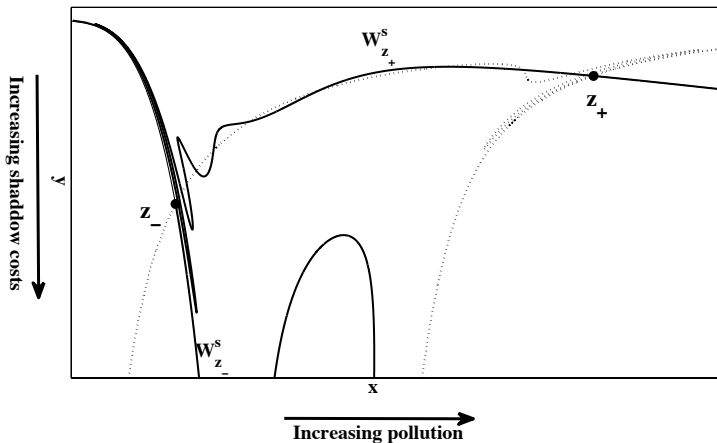
$(x_{t+1}, y_{t+1}) = \varphi(x_t, y_t)$:

$$\begin{cases} x_{t+1} = h(x_t) - \frac{h'(x_t)}{y_t + 2c x_t} \\ y_{t+1} = \frac{y_t + 2c x_t}{h'(x_t)} e^\rho, \end{cases}$$

$$h(x) := (1 - b)x + \frac{x^q}{1 + x^q}$$

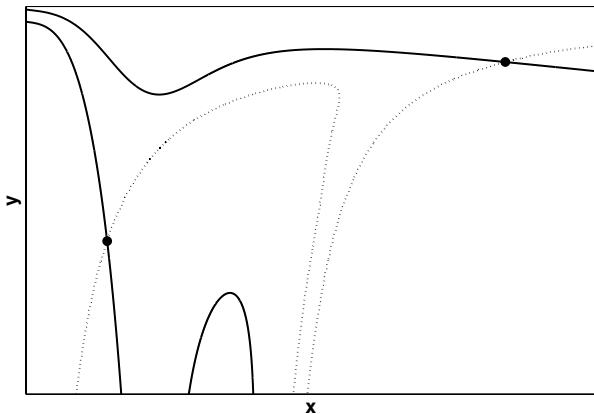
DLP phase diagram

$b = 0.6$, $\rho = 0.03$, $q = 4$ and $c_{ft} < c < c_{ea} < c_{lt}$



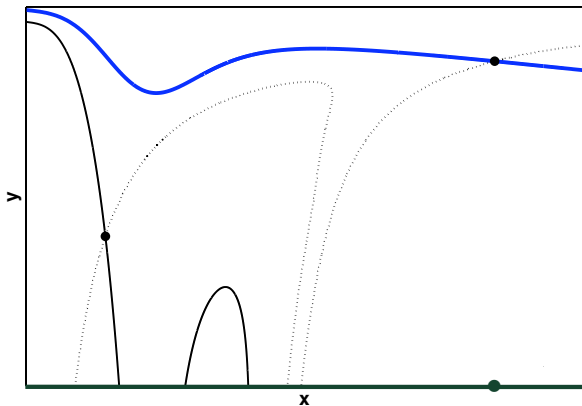
Unique globally attracting steady state

$$c = 0.14 < c_{ft}$$



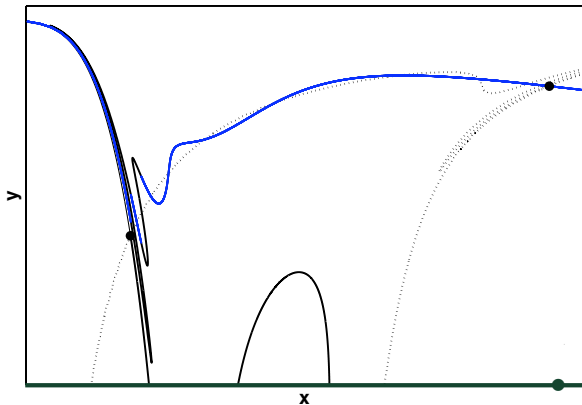
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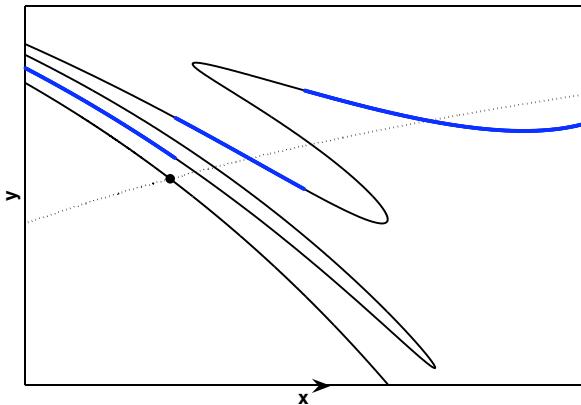
Genesis of indifference points

$$c_{ft} < c = 0.153 < c_{ea} < c_{lt}$$



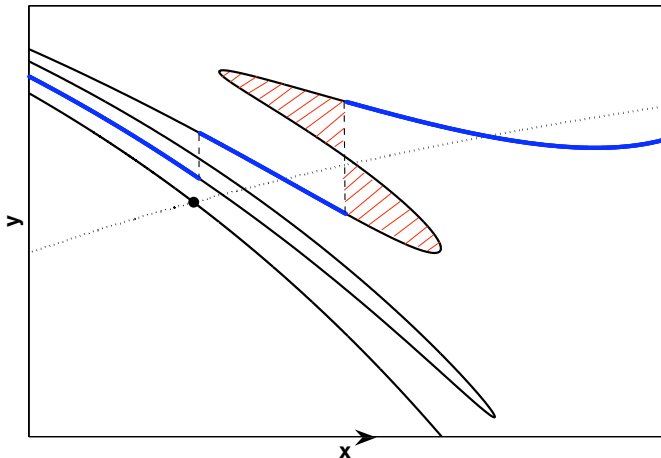
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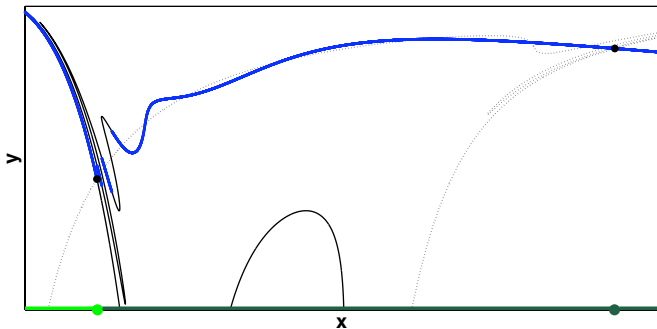
Unique globally attracting steady state and genesis of indifference points

$$c_{ft} < c = 0.153 < c_{ea} < c_{lt}$$



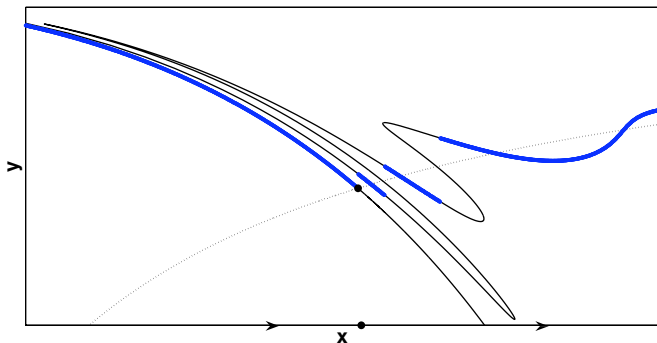
Semi-stable steady state

$$c_{ea} = c = 0.1541 < c_{lt}$$



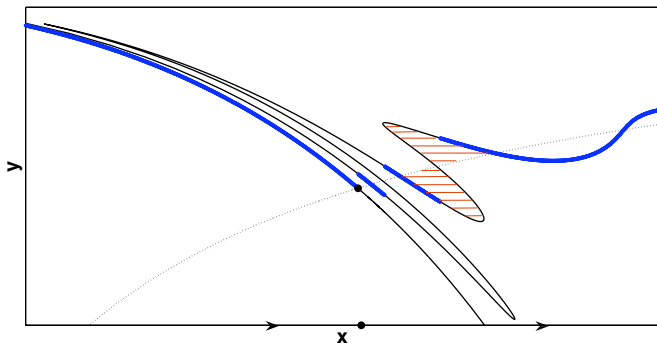
Semi-stable steady state and infinitely many indifference points

$$c_{ea} = c = 0.1541 < c_{lt}$$



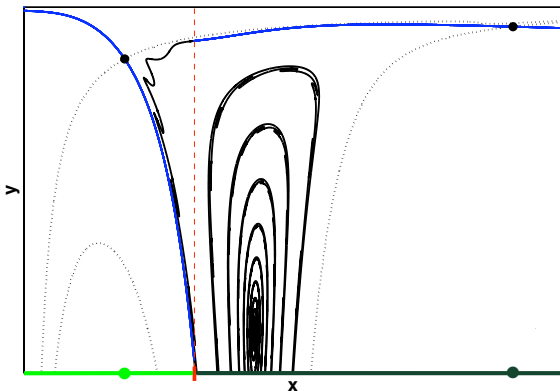
Semi-stable steady state and infinitely many indifference points

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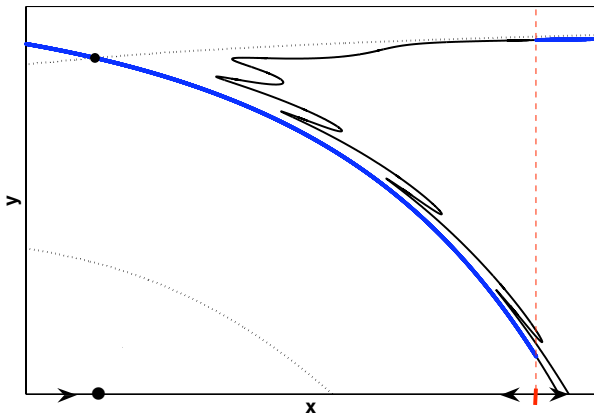
Genesis of indifference threshold

$$c_{ea} < c_{lt} < c = 0.17$$



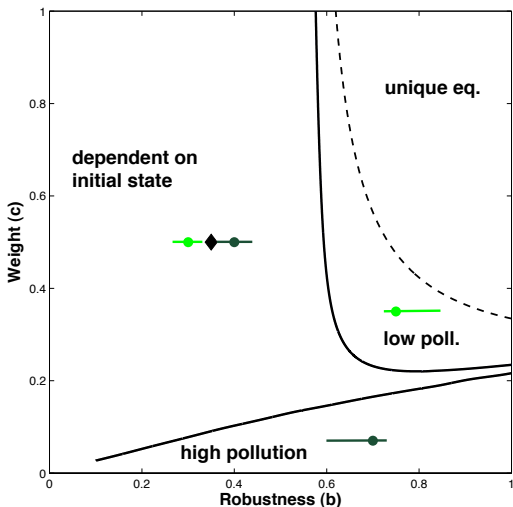
Genesis of indifference threshold

$$c_{ea} < c_{lt} < c = 0.17$$



Bifurcation Diagram

'weight of interests' (c) versus 'natural robustness' of the lake (b)



Lake pollution game: dynamic stock-dependent

- Stock-dependent discrete time dynamic games: payoffs and the stage game depend on an external evolving stock-variable, whose evolution (described by a discrete time dynamical system) in turn depends on the actions of players
- Illustration example: lake pollution model (Maler et al. (2003))
 - Dechert and Brock (2000) adapted their model to a discrete time model and computed numerical solutions for the optimal policy function
 - Saeed Moghayer (2011): Apply bifurcations of indifference points method to the model
 - Brock & de Zeeuw (2002): Repeated lake game
 - Dechert & Donnell, (2006): Stochastic (dynamic) lake game

Discrete time shallow lake model

Shallow lake dynamics

$$s_t = a_t + g(s_{t-1}), \quad s_0 \text{ is given,}$$

where

$$g(s) = (1 - b)s + \frac{s^q}{1 + s^q}$$

- $s_t \geq 0$: amount of phosphorus in a shallow lake;
- a_t : external loading of phosphorus;
- b : rate of loss of phosphorus;
- $\frac{s^q}{1 + s^q}$: biological production of phosphorus in the lake

2-player, 2-strategy dynamic stock-dependent game

- Two myopic players: farmers sharing the lake
- Two (pre-determined) actions: $a_t^i \in \{a_L, a_H\}$, $a_L < a_H$
- Payoffs: $\pi^i(a_t^i, s_t) = \log a_t^i - cs_t^2$, $i \in \{1, 2\}$
 - $\log a_t^i$: benefit to farmer i from using fertilizers
 - cs_t^2 : cost of pollution to each farmer

$$s_t = a_t^1 + a_t^2 + g(s_{t-1}),$$

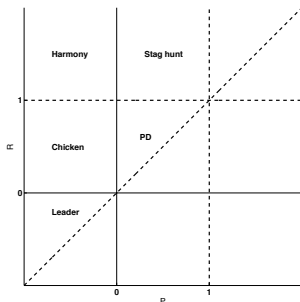
	low	high
low	$R(s_t)$	$S(s_t)$
high	$T(s_t)$	$P(s_t)$

- Markov stationary strategies: (pure) stage game Nash equilibrium in each period

Pay-off normalization:

	low	high
low	$R(s_t)$	0
high	1	$P(s_t)$

Classification of 2×2 games in (P, R) -space:



Fix parameters: $b = 0.6$, $q = 2$, $a_L = 0.1$, $a_H = 0.8$, $x_0 = 0.1$

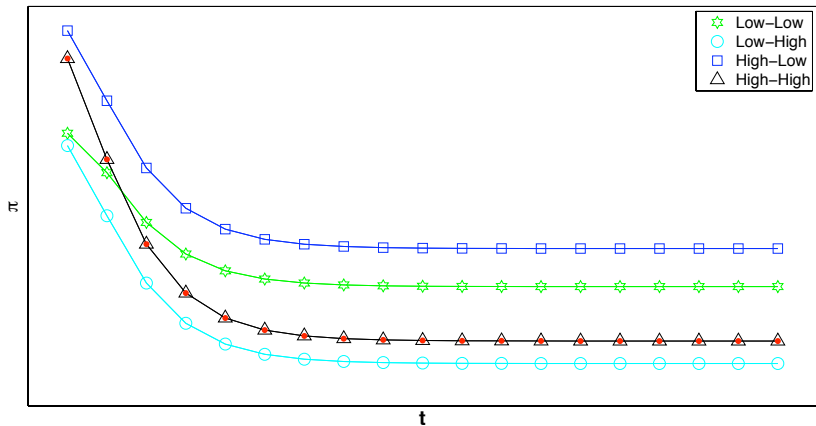


Figure: $c = 0.3$

Phase portrait in (P, R) -space for $c = 0.3$

(Loading Gmae dynamics $c=0.3$)

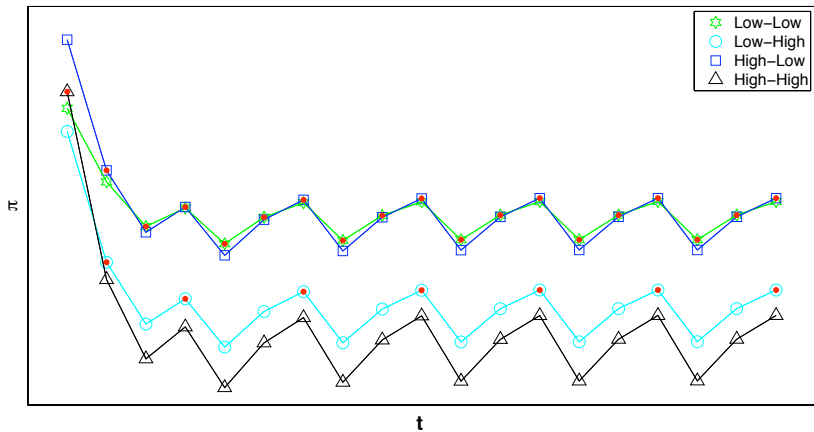


Figure: $c = 0.7$

Phase portrait in (P, R) -space for $c = 0.7$

(Loading Gmae dynamics $c=0.7$)

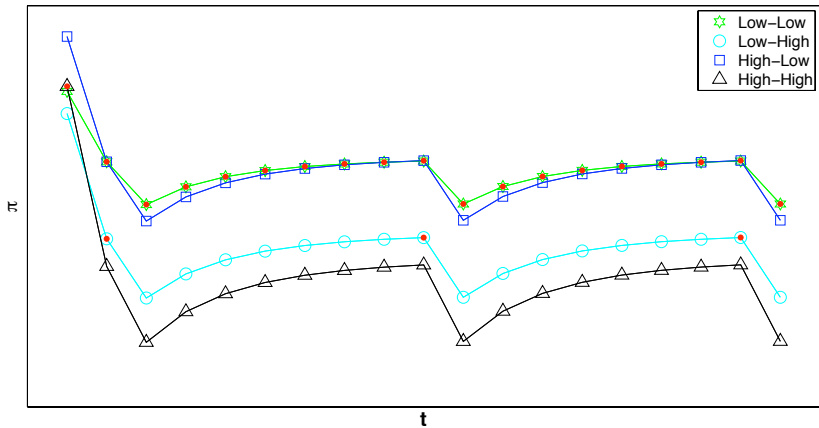


Figure: $c = 0.8$

Phase portrait in (P, R) -space for $c = 0.8$

(Loading Gmae dynamics $c=0.8$)

stock-dependent lake game with a continuous action space

Unique static Nash equilibrium at each date:

$$a_t^1 = a_t^2 = \frac{1}{4} \left(\sqrt{\bar{c} - (g(s_{t-1}))^2} \right), \quad t = 1, 2, \dots,$$

where $\bar{c} = 1/c$.

Lake dynamics:

$$s_t = \frac{1}{2} \left(\sqrt{\bar{c} + (g(s_{t-1}))^2} + g(s_{t-1}) \right), \quad s_0 \text{ is given}$$

Fold bifurcation ($q = 4$)

